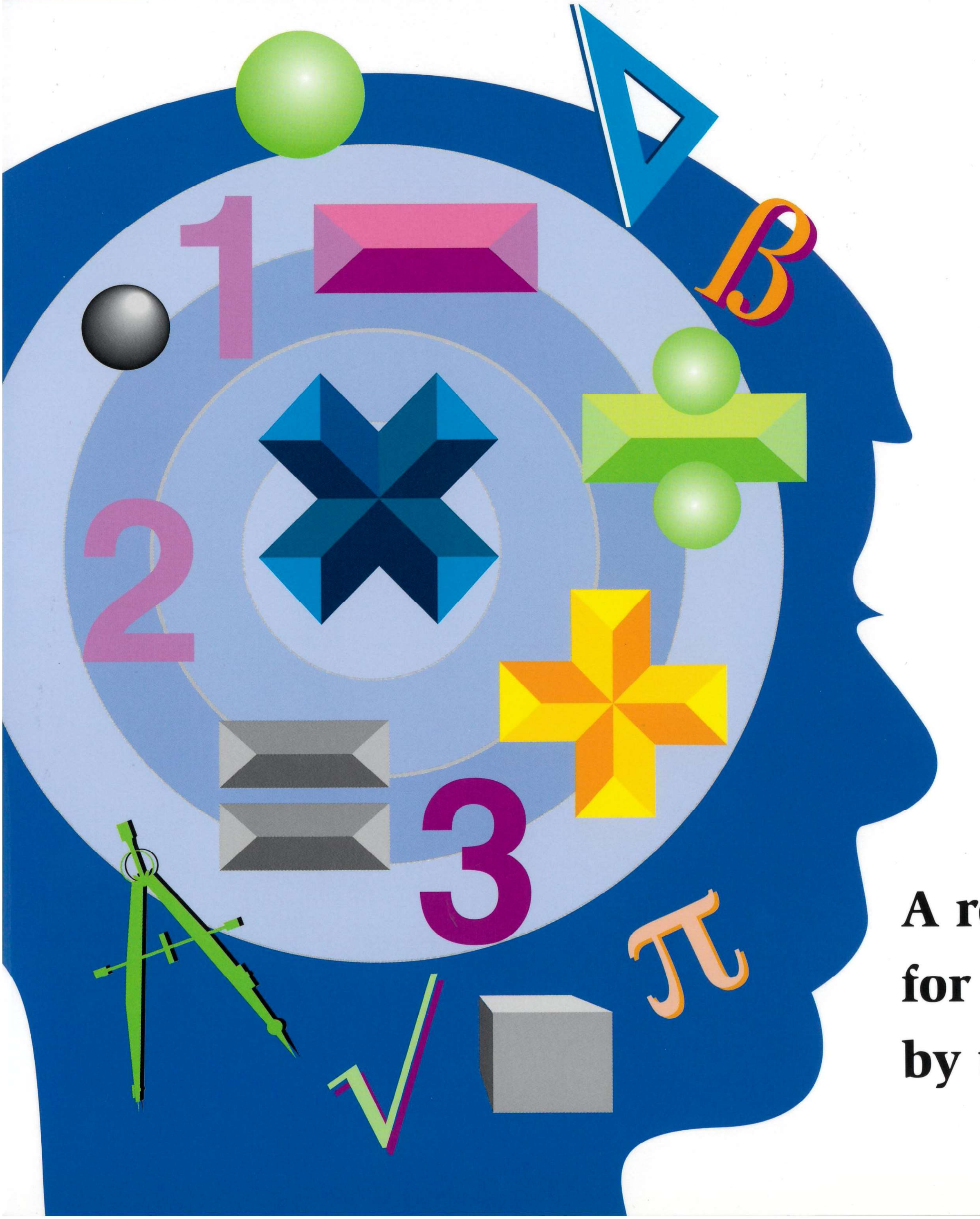
Pathways to reasoning and communication in the primary school mathematics classroom



A resource for teachers by teachers

Pathways to reasoning and communication in the primary school mathematics classroom

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FOREWORD

The timely publication of *Pathways to Reasoning and Communication in the Primary School Mathematics Classroom* highlights and exemplifies two critical developments in the professional development of teachers in many parts of the world over the past decade or so.

The first is a broad recognition that success in mathematics depends as much on achieving high levels of performance in mathematical reasoning and communication as it does on mastery of basic factual information, skills and procedures. There is little doubt, given the exceptional performance of Singaporean students on TIMSS assessments, that mathematics teaching in Singapore has achieved world class standards in the development of key mathematical skills and procedural fluency. However, there is a widespread conviction – and some evidence – that Singaporean students are not as adept as they might be in more complex forms of mathematical reasoning and communication of the kind that knowledge economy work places, for example, require of students. *Pathways to Reasoning and Communication in the Primary School Mathematics Classroom* is designed to address this challenge.

The second development reflects a gradual shift in the centre of gravity away from the University-based, "supply-side," "off-line" forms knowledge production conducted by university researchers for teachers towards an emergent school-based, demand-side, on-line, *in situ* forms of knowledge production conducted by teachers for teachers. Supporters of this transition do not deny the value of university based research, but they do insist that in a knowledge economy, improving the quality of teaching and learning is going to depend increasingly on carefully crafted partnerships between university scholars and classroom teachers. Critically, they also insist that one key outcome of such partnerships ought to be the codification, verification, dissemination and institutionalization of expert teacher knowledge. *Pathways to Reasoning and Communication in the Primary School Mathematics Classroom* exemplifies this partnership.

For these reasons and more, I warmly welcome and congratulate my colleagues here in NIE and their teacher collaborators in the schools for the publication of *Pathways to Reasoning and Communication in the Primary School Mathematics Classroom.* It not only reflects a fruitful partnership between NIE researchers and school teachers over the past several years, but is also a harbinger of one key component of the future of pedagogical knowledge production and management in Singapore.

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A Special Note To Teachers

The 2007 revised framework for school mathematics curriculum places emphasis on reasoning, communication and connections in addition to thinking skills and heuristics. As thinking skills and heuristics have been the processes teachers are comfortable with in their classrooms for a long while, the intention of the project Enhancing the Pedagogy of Mathematics Teachers (EPMT) was to engage teachers in adapting tasks from their textbooks and other sources for use in their lessons so as to facilitate reasoning and communication amongst their students. The EPMT project is an intervention type of project funded by the Centre for Research in Pedagogy and Practice at the National Institute of Education in Singapore.

This resource book is a product of the EPMT (Primary) project. It contains the numerous strategies that the 20 participants from 5 primary schools were exposed to by A/P Berinderjeet Kaur, the principal investigator of the project, during the first professional development phase of the project. The task sheets accompanying each strategy showcases the work of teachers in the project. They are meant for fellow mathematics teachers to adopt/adapt and use in their classrooms.

We hope this small contribution will go a long way towards making every mathematics classroom a venue for much thought and reasoning where pupils use mathematical language to express themselves.

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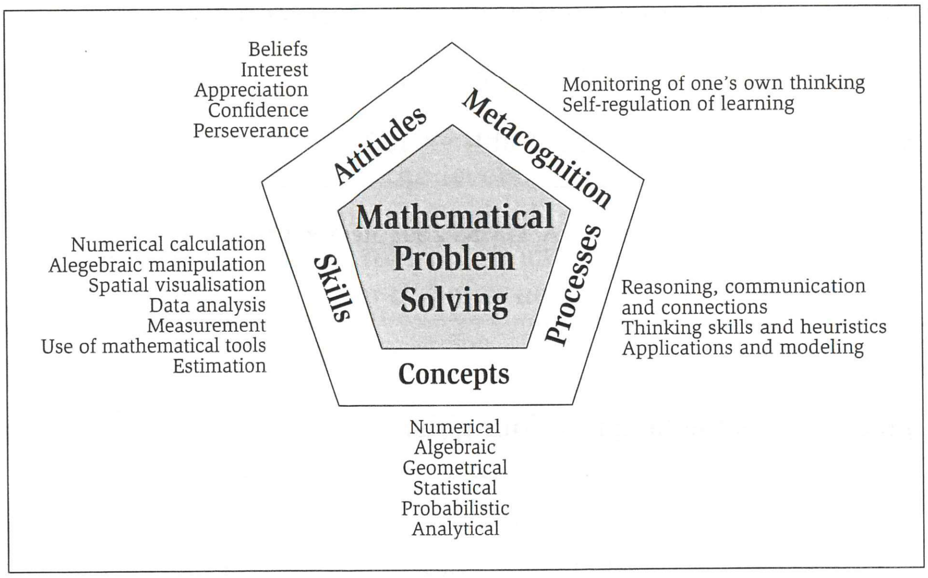
Introduction

Why Reasoning and Communication in the Primary School Mathematics Classroom

The revised framework for school mathematics in Singapore implemented in 2007 by the Ministry of Education (MOE), has expanded the scope of Processes to include

- **Reasoning, communication and connections**
- Thinking skills and heuristics
- Applications and modeling



Framework of the School Mathematics Curriculum

Teachers are familiar with thinking skills and heuristics as both have been apart of the framework for the last decade. As mathematical reasoning, communication and connections are new attributes of the framework implemented in 2007, there was a need to work with teachers in this area.

In the Mathematics Syllabus (Ministry of Education, 2006) for primary school, the scope of reasoning, communication and connections has been outlined as follows:

- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments. It is a habit of mind that can be developed through the application of mathematics in different situations and contexts.

- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps students develop their own understanding of mathematics and sharpens their mathematical thinking.

- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and everyday life. This helps students make sense of what they learn in mathematics.

What is reasoning?

Reasoning is the ability to think, understand, and form opinions or judgments that are based on facts (Longman, 1987). It is the process of making inferences from a body of information. For example, given the information that Lala is a spider, it is reasonable to conclude that Lala has eight legs. Likewise, given the information that Sally's room is a rectangle 4 m wide and 6 m long, it is reasonable to conclude that the area of the room is 24 sq m. Figures 1 and 2 show two items from TIMSS 2003 with the performance expectation - reasoning.

Item M09/02

Juanita wanted to use her calculator to add 1379 and 243. She entered 1279 + 243 by mistake. Which of these could she do to correct her mistake?

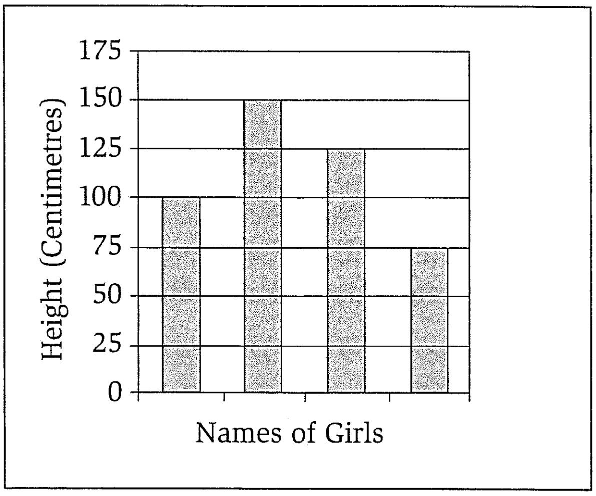
A. Add 100 C. Subtract 1

B. Add 1 D. Subtract 100

Figure 1: A TIMSS 2003 Item

Item M03/01

The graph shows the height of four girls.



The names are missing from the graph. Debbie is the tallest. Amy is the shortest. Dawn is taller than Sarah. How tall is Sarah?

A. 75 cm B. 100 cm C. 125 cm D. 150 cm

Figure 2: A TIMSS 2003 Item

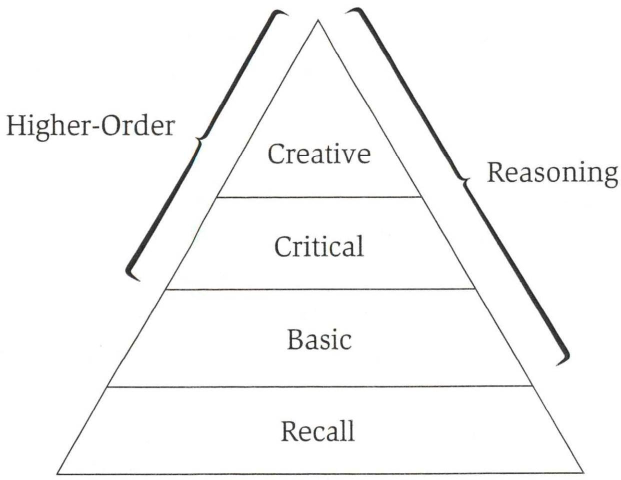


Figure 3: Hierarchy of Thinking (Krulik & Rudnick, 1993)

Figure 3 shows the building blocks of Krulik and Rudnick's (1993) hierarchy of thinking. As thinking is a complex process the categories shown are not discrete and each level makes extensive use of the skills contained in the levels that lie below it. This hierarchy is based on the premise that thinking is the ability of a child to reach a valid conclusion from a given set of data. The child must make conjectures, abstract properties from relationships in problem situations, then validate and explain his or her conclusions or assertions.

Recall thinking includes thinking skills that are almost automatic or reflexive in nature. This might include recall of basic number facts, such as $3 + 4 = 7$, $2 \times 5 = 10$; conversions such as 100 cm = 1 m or algorithms such as area of a rectangle = length × breadth. This form of thinking is synonymous to "remembering" in the revised Bloom's taxonomy of cognitive goals, (Anderson & Krathwohl, 2001) shown in figure 4.

Basic thinking includes the understanding of mathematical concepts as well as the recognition of their application in textbook and everyday problems. An example of basic thinking would be finding the total cost of 8 bars of chocolates at $1.20 each and the change to expect from a cashier when paying with a $10 note. This form of thinking is synonymous to "understanding" and "applying" in the revised Bloom's taxonomy of cognitive goals (Anderson & Krathwohl, 2001), shown in figure 4. The recall and basic forms of thinking are complimentary in primary classrooms as much of the mathematics taught at the primary level requires pupils to be proficient in basic knowledge, comprehend the concepts taught and apply them to solve textbook and everyday problems.

Critical thinking is thinking that examines, relates, and evaluates all aspects of the situation or problem. It is analytical and reflexive in nature. Hence critical thinking includes the following:

- the ability to read with understanding,
- the ability to identify necessary and extraneous information,
- the ability to recognize what is asked or required,
- the ability to recognize that there is insufficient data or even contradictory data, and
- the ability to determine the reasonableness of an answer (Krulik & Rudnick; 1993, 1999).

This form of thinking is synonymous to "analyzing"and “evaluating” in the revised Bloom's taxonomy of cognitive goals (Anderson & Krathwohl, 2001), shown in figure 4.

Creative thinking is thinking that is original and reflective and that produces a complex product. It includes synthesizing ideas, generating new ideas, and determining their effectiveness. Creative thinking includes the ability to make decisions and usually culminates in the production of a new end product (Krulik & Rudnick, 1993). This form of thinking is synonymous to “creating” in the revised Bloom's taxonomy of cognitive goals (Anderson & Krathwohl, 2001), shown in figure 4.

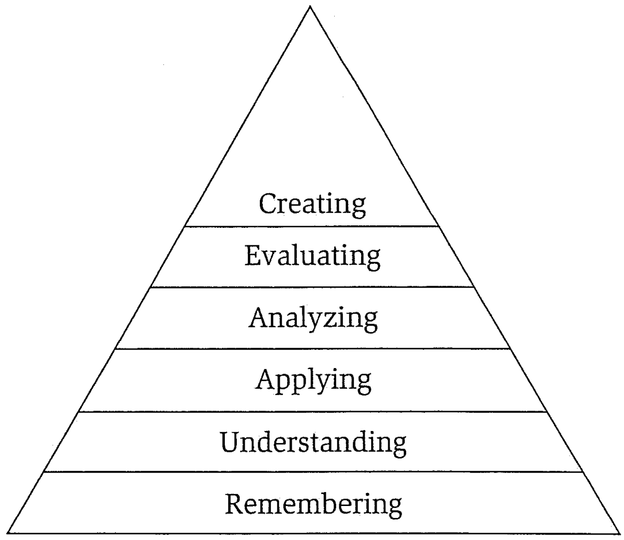


Figure 4: Revised Bloom’s taxonomy of cognitive goals

Strategies For Reasoning And Communication

Inductive Reasoning

Inductive reasoning is the process of arriving at a conclusion based on a set of observations. It is a method of reasoning from particular to general, i.e., the mental process involved in creating generalisations from observations.

Inductive arguments can include:

- Part-to-whole: where the whole is assumed to be like individual parts (only bigger).
- Extrapolations: where areas beyond the area of study are assumed to be like the studied area.
- Predictions: where the future is assumed to be like the past.

Common examples in primary mathematics include:

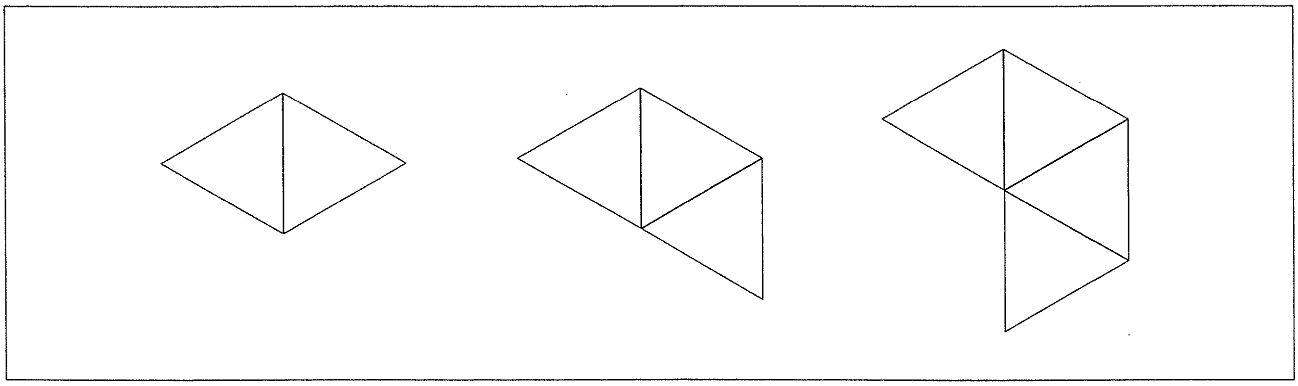
EXAMPLE 1

Find the missing numbers in this sequence 1265, 1275, 1285, _______, _______, 1315, 1325,...

By considering the first three terms, one observes that the digit in the tens place increases in a certain way. If a teacher asks pupils to find the 50th term without writing all of them down then the pupils need to derive a general rule.

EXAMPLE 2

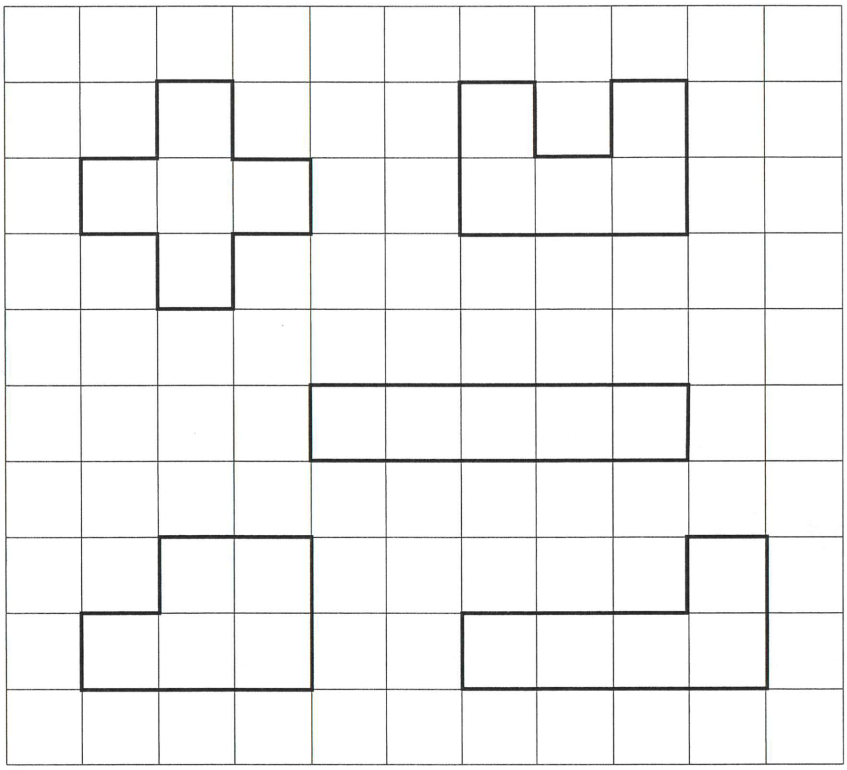
By cutting each shape into as few triangles as possible, find the sum of the angles in the shape.



Number of sides	Number of triangles	Sum of angles
4	2	2 x 180°
5	3	3 x 180°
6	4	4 x 180°
7	5	5 x 180°
8	6	6 x 180°

What is the sum of angles in a 50 sided shape?
What is the sum of angles in an n sided shape?

EXAMPLE 3



Find the area and perimeter of the following shapes.

Using inductive reasoning, pupils conclude that shapes that have the same area may not neccessarily have the same perimeter.

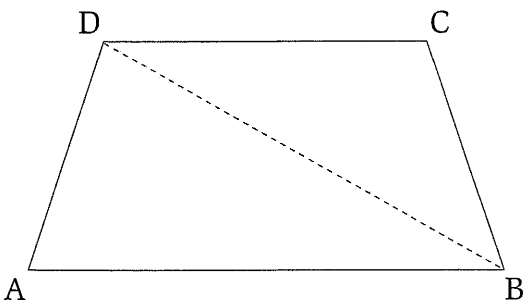
Deductive Reasoning

Deductive reasoning, or deduction, starts with a general case and deduces specific instances. Deduction is used by scientists who take a general scientific law and apply it to a certain case. Deductive reasoning assumes that the basic law from which you are arguing is applicable in all cases. This can let you take a rule and apply it perhaps where it was not really meant to be applied.

Common examples in primary mathematics include:

EXAMPLE 1

Find the sum of angles in a trapezium.

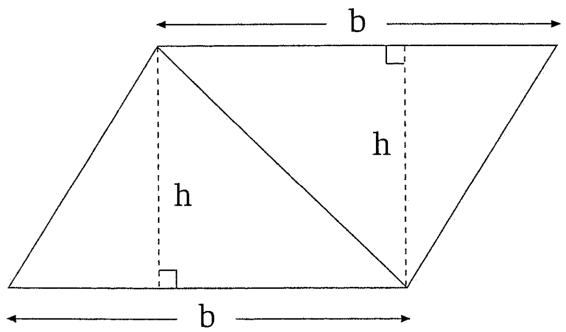


Using the general rule that the sum of angles in a triangle is 180°, pupils use deductive reasoning to conclude that the sum of angles in a trapezium is 360°.

EXAMPLE 2

Find the area of a parallelogram.

Using the area of a triangle $= \frac{1}{2}$ x base x height and the properties of a parallelogram, pupils can deduce that the area of parallelogram = base x height.



Area of parallelogram

$= \frac{1}{2}(b \times h) + \frac{1}{2}(b \times h)$

$= b \times h$

= base x height